Bianchi Type-I Cosmological Models with Variable G and Λ-Term in General Relativity

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Abstract

Einstein's field equations with variable gravitational and cosmological "constant" are considered in presence of perfect fluid for Bianchi type-I space-time. Consequences of the four cases of the phenomenological decay of Λ have been discussed which are consistent with observations. The physical significance of the cosmological models have also been discussed.

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1 Introduction

There are significant observational evidence that the expansion of the Universe is undergoing a late time acceleration (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 2004; Efstathiou et al. 2002; Spergel et al. 2003; Allen et al. 2004; Sahni and Starobinsky 2000; Peebles and Ratra 2003; Padmanabhan 2003; Lima 2004). This, in other words, amounts to saying that in the context of Einstein's general theory of relativity some sort of dark energy, constant or that varies only slowly with time and space dominates the current composition of cosmos. The origin and nature of such an accelerating field poses a completely open question. The main conclusion of these observations is that the expansion of the universe is accelerating.

Among many possible alternatives, the simplest and most theoretically appealing possibility for dark energy is the energy density stored on the vacuum state of all existing fields in the universe, i.e., $\rho_v = \frac{\Lambda}{8\pi G}$, where Λ is the cosmological constant. However, a constant Λ cannot explain the huge difference between the cosmological constant inferred from observation and the vacuum

energy density resulting from quantum field theories. In an attempt to solve this problem, variable Λ was introduced such that Λ was large in the early universe and then decayed with evolution (Dolgov 1983). Cosmological scenarios with a time-varying Λ were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during last two decades (Chen and Hu 1991; Pavon 1991; Carvalho, Lima and Waga 1992; Lima and Maia 1994; Lima and Trodden 1996; Arbab and Abdel-Rahaman 1994; Vishwakarma 2001, Cunha and Santos 2004; Carneiro and Lima 2005).

On the other hand, numerous modifications of general relativity to allow for a variable G based on different arguments have been proposed (Wesson 1980). Variation of G has many interesting consequences in astrophysics. Canuto and Narlikar (1980) have shown that G-varying cosmology is consistent with whatsoever cosmological observations available at present. A modification linking the variation of G with that of variable Λ-term has been considered within the framework of general relativity by a number of workers (Kallingas et al. 1992; Abdel-Rahaman 1990; Berman 1991; Beesham 1986). This modification is appealing as it leaves the form of Einstein's equations formally unchanged by allowing a variation of G to be accompanied by a change in Λ. Cosmological models with time-dependent G and Λ in the solutions $\Lambda \sim R^{-2}, \Lambda \sim t^{-2}$, were first obtained by Bertolami (1986). The cosmological models with variable G and Λ have been recently studied by several authors (Arbab 2003; Sistero 1991; Sattar and Vishwakarma 1997; Pradhan et al., 2001, 2002, 2005, 2007; Singh et al., 2006, 2007).

Another important quantity which is supposed to be damped out in the course of cosmic evolution is the anisotropy of the cosmic expansion. Theoretical arguments and recent experimental data support the existence of an anisotropic phase that approaches an isotropic one. Therefore, it makes sense to consider the models of the universe with anisotropic background in the presence of dark energy.

The simplest of anisotropic models are Bianchi type-I homogeneous models whose spatial sections are flat but the expansion or contraction rate are direction dependent. For studying the possible effects of anisotropy in the early universe on present day observations many researchers (Huang 1990; Chimento et al. 1997; Lima 1996; Lima and Carvalho 1994; Pradhan et al. 2004, 2006; Saha 2005, 2006) have investigated Bianchi type-I models from different point of view.

In the present article, we present a new class of solutions to Einstein's field equations with variable G and Λ in Bianchi type-I space-time in the presence of a perfect fluid. Consequences of the following four cases of the phenomenological decay of Λ have been discussed:

$$Case\ 1 : \Lambda \sim H^2,$$

$$Case\ 2 : \Lambda \sim H,$$
$$Case\ 3 : \Lambda \sim \rho,$$
$$Case\ 4 : \Lambda \sim R^{-2},$$

where H, ρ, R are respectively the Hubble parameter, energy density and average scale factor of the Bianchi type-I metric. The dynamical laws proposed for the decay of Λ have been widely studied by Chen and Wu (1990), Carvalho et al. (1992), Schutzhold (2002), Vishwakarma (2000), Arbab (1997, 1998) to name only a few.

2 The Metric, Field Equations and Solutions

We consider the space-time admitting Bianchi type-I group of motion in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2. \tag{1}$$

We assume that the cosmic matter is represented by the energy-momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}, \tag{2}$$

where ρ, p are the energy density, thermodynamical pressure and v_i is the four-velocity vector of the fluid satisfying the relation

$$v_i v^i = -1. \tag{3}$$

The Einstein's field equations with time-dependent G and Λ are

$$R_{ij} - \frac{1}{2}R g_{ij} = -8\pi G(t) T_{ij} + \Lambda(t) g_{ij}. \tag{4}$$

For the metric (1) and energy-momentum tensor (2) in comoving system of coordinates, the field equation (4) yields

$$8\pi G p - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \tag{5}$$

$$8\pi G p - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC}, \tag{6}$$

$$8\pi G p - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB}, \tag{7}$$

$$8\pi G \rho + \Lambda = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}. \tag{8}$$

In view of vanishing divergence of Einstein tensor, we get

$$8\pi G\Big[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\Big] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{9}$$

The usual energy conservation equation $T^{j}_{i;j} = 0$ yields

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{10}$$

Equation (9) together with (10) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{11}$$

Here and elsewhere a dot stands for ordinary time-derivative of the concerned quantity. From equation (11) one concludes that when Λ is constant or $\Lambda = 0$, G turns out to be constant.
Let R be the average scale factor of Bianchi type-I universe i.e.

$$R^3 = \sqrt{-g} = ABC. \tag{12}$$

From equations (5), (6) and (7), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3}, \tag{13}$$

and

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3}. \tag{14}$$

On integration equations (13) and (14) give

$$A = m_1 \; R \quad \exp\left[\frac{(2k_1 + k_2)}{3} \int \frac{dt}{R^3}\right],$$

$$B = m_2 \; R \quad \exp\left[\frac{(k_2 - k_1)}{3} \int \frac{dt}{R^3}\right],$$

$$C = m_3 \; R \quad \exp\left[-\frac{(k_1 + 2k_2)}{3} \int \frac{dt}{R^3}\right], \tag{15}$$

where k_1, k_2, m_1, m_2, m_3 are arbitrary constants of integration satisfying

$$m_1 m_2 m_3 = 1.$$

Similar expressions as (15) have also been established by Saha (2005).

Hubble parameter H, volume expansion θ, shear σ and deceleration parameter q are given by

$$\theta = 3H = 3\frac{\dot{R}}{R},$$

$$\sigma = \frac{k}{\sqrt{3}R^3}, \quad k > 0, \text{(constant)}$$

$$q = -1 - \frac{\dot{H}}{H^2}.$$

Equations (5)-(8) and (10) can be written in terms of H, σ and q as

$$8\pi Gp = H^2(2q-1) - \sigma^2 + \Lambda, \tag{16}$$

$$8\pi G\rho = 3H^2 - \sigma^2 - \Lambda, \tag{17}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0. \tag{18}$$

It is to note that energy density of the universe is a positive quantity. It is believed that at the early stages of the evolution when the average scale factor R was close to zero, the energy density of the universe was infinitely large. On the other hand, with the expansion of the universe i.e. with increase of R, the energy density decreases and an infinitely large R corresponds to a ρ close to zero. In that case from (17), we obtain

$$3H^2 - \Lambda \to 0. \tag{19}$$

From equation (19) one concludes that:

(i) Λ is essentially non-negative,

(ii) in absence of a Λ-term beginning from some value of R, the evolution of the universe becomes stand-still i.e. R becomes constant since H becomes zero,

(iii) in case of a positive Λ, the process of evolution of the universe never comes to a halt. Moreover, it is believed that the presence of dark energy (given by positive Λ) results in the accelerated expansion of the universe. As far as negative Λ is concerned, its presence imposes some restriction on ρ i.e ρ can never be small enough to be ignored. It means, in that case there exists some upper limit for R as well. It is worth mention here that Saha (2006) has also given such conclusion in his paper but his approach was quite different.
From equation (17), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}.$$

Therefore, $0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3}$ and $0 \leq \frac{8\pi G\rho}{\theta^2} \leq \frac{1}{3}$ for $\Lambda \geq 0$.

Thus, the presence of a positive Λ puts restriction on the upper limit of anisotropy whereas a negative Λ contributes to the anisotropy.

From equation (16), we obtain

$$\frac{d\theta}{dt} = -\frac{3}{2}\{8\pi Gp + 3H^2 - \Lambda + \sigma^2\}$$

Thus for negative Λ, the universe will always be in decelerating phase whereas a positive Λ will slow down the rate of decrease. Also $\sigma_4 = -3\sigma H$ implying that σ decreases in an evolving universe and for infinitely large value of R, σ becomes negligible.

Equations (5) - (8) and (11) together with one of the decay laws for Λ given by cases (1) - (4) supply six equations in seven unknown functions of time A, B, C, ρ, p, Λ and G. To have deterministic solutions, we require one more condition. For this purpose, we assume that the volume expansion θ is proportional to eigen values of shear tensor σ_{ij}. It is believed that evolution of one parameter should also be responsible for the evolution of the others (Vishwakarma, 2005). Following Roy and Singh (1985), we take the volume expansion θ having a constant ratio to the anisotropy in the direction of unit space-like vector λ^i i.e. $\frac{\theta}{\sigma_{ij}\lambda^i\lambda^j}$ is constant. In general, the above condition gives rise to

$$A = B^m C^n, \tag{20}$$

where m and n are constants. Using condition (20) in equations (13) and (14), we obtain

$$\begin{aligned} C &= b_1(k_3 t + k_4)^{\frac{k_1-(m-1)k_2}{(m+n+2)k_1-(m-2n-1)k_2}} \quad for \quad \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2}, \\ &= k_5 \ \exp\left[\frac{-k_2(m+1)t}{m+n+2}\right] \quad for \quad \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2}, \end{aligned} \tag{21}$$

$$\begin{aligned} B &= b_2(k_3 t + k_4)^{\frac{k_1+k_2 n}{(m+n+2)k_1-(m-2n-1)k_2}} \quad for \quad \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2}, \\ &= b_3 \ \exp\left[\frac{k_2(n+1)t}{m+n+2}\right] \quad for \quad \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2}, \end{aligned} \tag{22}$$

provided $m + n \neq 1$. In the above k_3, k_4, k_5 and b_1, b_2, b_3 are constants of integration. For these solutions, metric (1) takes the following forms after suitable transformations:

$$\begin{aligned} ds^2 = -dT^2 &+ T^{\frac{2(m+n)k_1+2nk_2}{(m+n+2)k_1-(m-2n-1)k_2}} dX^2 + T^{\frac{2k_1+2nk_2}{(m+n+2)k_1-(m-2n-1)k_2}} dY^2 \\ &+ T^{\frac{2k_1-2(m-1)k_2}{(m+n+2)k_1-(m-2n-1)k_2}} dZ^2 \quad for \quad \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2}, \end{aligned} \tag{23}$$

and

$$\begin{aligned} ds^2 = -dT^2 + \exp\left[\frac{2k_2(m-n)T}{m+n+2}\right] dX^2 + \exp\left[\frac{2k_2(n+1)T}{m+n+2}\right] dY^2 + \\ \exp\left[-\frac{2k_2(m+1)T}{m+n+2}\right] dZ^2 \quad for \quad \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2}. \end{aligned} \tag{24}$$

3 Discussion

We now describe the models resulting from different dynamical laws for the decay of Λ.

For the model (23), average scale factor R is given by

$$R = T^{\frac{1}{3}}.$$

Volume expansion θ, Hubble parameter H and shear σ for the model are:

$$\theta = 3H = \frac{1}{T}, \quad \sigma^2 = \frac{k^2}{3T^2}.$$

Thus we see that $\frac{\sigma}{\theta} = \frac{k}{\sqrt{3}}$. Therefore, the model does not approach isotropy. If k is small, the models are quasi-isotropic i.e. $\frac{\sigma}{\theta} = 0$.

3.1 Case 1 :

We consider

$$\Lambda = 3\beta H^2,$$

where β is a constant of the order of unity. Here β represents the ratio between vacuum and critical densities. From equations (5), (8) and (11), we obtain

$$8\pi\rho = \frac{(1-k^2-\beta)}{3k_0} T^{-\frac{2(1-k^2)}{(1-k^2-\beta)}}, \tag{25}$$

$$8\pi p = \frac{(1-k^2+\beta)}{3k_0} T^{-\frac{2(1-k^2)}{(1-k^2-\beta)}}, \tag{26}$$

$$\Lambda = \frac{\beta}{3T^2}, \tag{27}$$

$$G = k_0 T^{\frac{2\beta}{(1-k^2-\beta)}}, \quad k_0 > 0(\text{constant}). \tag{28}$$

We observe that the model has singularity at $T = 0$. It starts with a big bang from its singular state and continues to expand till $T = \infty$. At $T = 0$, ρ, p, Λ, θ and σ are all infinite whereas $G = 0$ for $\beta > 0$ and $G = \infty$ for $\beta < 0$. For infinitely large T, ρ, p, Λ, θ and σ are all zero but $G = \infty$ for $\beta > 0$ and $G = 0$ for $\beta < 0$. We also observe that in the absence of cosmological term $\Lambda(\beta = 0)$, $\rho = p$ i.e. matter content turns out to be a stuff fluid. For $\beta > 0$, $p > \rho$ and $p < \rho$ when $\beta < 0$. When $\beta = k^2 - 1$, $p = 0$. The density parameter $\Omega = \frac{\rho}{\rho_c} = 1 - k^2 - \beta$ implying that $\rho_c > \rho$ and $\rho_c < \rho$ for $\beta > -k^2$ and $\beta < -k^2$ respectively whereas $\rho_c = \rho$ when $\beta = -k^2$. The ratio between vacuum and matter densities is given by

$$\frac{\rho_v}{\rho} = \frac{\beta}{1 - k^2 - \beta}.$$

3.2 Case 2 :

We now consider

$$\Lambda = aH,$$

where a is a positive constant of order of m^3 where $m \approx 150MeV$ is the energy scale of chiral phase transition of QCD (Borges and Carneiro, 2005). For this case, equations (5), (8) and (11) yield

$$8\pi\rho = \frac{(1-k^2-aT)^2}{3k_0T^2}, \tag{29}$$

$$8\pi p = \frac{(1-k^2)^2 - a^2T^2}{3k_0T^2}, \tag{30}$$

$$\Lambda = \frac{a}{3T}, \tag{31}$$

$$G = \frac{k_0}{1-k^2-aT}. \tag{32}$$

The model has singularity at $T = 0$. The model starts from a big bang with ρ, p, Λ, θ, σ all infinite and G finite. Thereafter ρ, p, Λ, θ and σ decrease and G increases. When $T = \frac{1-k^2}{a}$, we obtain $p = 0$, $\rho = 0$, $\Lambda = \frac{a^2}{3(1-k^2)}$, $\sigma = \frac{ka}{\sqrt{3}(1-k^2)}$ and G is infinite. As $T \to \infty$, $\rho \sim \frac{a^2}{24\pi k_0}$, $p \sim -\frac{a^2}{24\pi k_0}$, and θ, σ, G, Λ tend to zero. The density parameter $\Omega = 1 - k^2 - aT$ and the ratio between vacuum and critical densities is given by

$$\frac{\rho_v}{\rho_c} = aT.$$

3.3 Case 3 :

We now consider

$$\Lambda = \frac{8\pi\alpha G\rho}{3},$$

where α is a constant. In this case from equations (5), (8) and (11), we obtain

$$8\pi\rho = \frac{(1-k^2)}{k_0(\alpha+3)} T^{-\frac{2(\alpha+3)}{3}}, \quad \alpha \neq -3, \tag{33}$$

$$8\pi p = \frac{(1-k^2)(2\alpha+3)}{k_0(\alpha+3)} T^{-\frac{2(\alpha+3)}{3}}, \tag{34}$$

$$\Lambda = \frac{\alpha(1-k^2)}{3(\alpha+3)T^2}, \tag{35}$$

$$G = k_0 T^{\frac{2\alpha}{3}}. \tag{36}$$

This model also starts from a big bang at $T=0$ with ρ, p, Λ, θ, σ all infinite and $G=0$ (for $\alpha>0$) and it evolves to $\rho\to 0$, $p\to 0$, $\theta\to 0$, $\sigma\to 0$, $\Lambda\to 0$ and $G\to\infty$ as $T\to\infty$. The density parameter Ω for this model is given by

$$\Omega=\frac{3(1-k^2)}{\alpha+3},$$

and the ratio between vacuum and critical densities is obtained as

$$\frac{\rho_v}{\rho_c}=\frac{\alpha(1-k^2)}{\alpha+3}.$$

3.4 Case 4 :

Finally we consider the case

$$\Lambda=\frac{\gamma}{R^2},$$

where γ is a parameter to be determined from the observations. In this case from equations (5) - (8) and (11), we obtain

$$8\pi\rho=\frac{[(1-k^2)T^{-\frac{4}{3}}-3\gamma]^{\frac{3}{2}}}{3k_0}, \tag{37}$$

$$8\pi p=\frac{[(1-k^2)T^{-\frac{4}{3}}+3\gamma]^{\frac{3}{2}}}{3k_0}, \tag{38}$$

$$\Lambda=\gamma T^{-\frac{2}{3}}, \tag{39}$$

$$G=k_0[1-k^2-3\gamma T^{\frac{4}{3}}]^{-\frac{1}{2}}. \tag{40}$$

Here we observe that this model also has singularity at $T=0$. It starts from a big bang singularity with ρ, p, θ, Λ, σ all infinite but G finite. For $\Lambda>0$ i.e. $\gamma>0$, ρ becomes zero at $T=\left(\frac{1-k^2}{3\gamma}\right)^{\frac{3}{4}}$ whereas for $\Lambda<0$ i.e. $\gamma<0$, $p=0$ at $T=\left(\frac{k^2-1}{3\gamma}\right)^{\frac{3}{4}}$. As $T\to\infty$, θ, σ, Λ and G become zero but ρ and p become finite. The density parameter Ω for this model is given by

$$\Omega=1-k^2-3\gamma T^{\frac{4}{3}}.$$

The ratio between vacuum and critical densities is given by

$$\frac{\rho_v}{\rho_c}=3\gamma T^{\frac{4}{3}}.$$

The model (24) is not of much interest since it reduces to a static solution.

4 Conclusion

In this paper, we have presented a class of solutions to Einstein's field equations with variable G and Λ in Bianchi type-I space-time in the presence of a perfect fluid. In some cases, it is observed that G is an increasing function of time. When the universe is required to have expanded from a finite minimum volume, the critical density assumption and conservation of energy-momentum tensor dictate that G increases in a perpetually expanding universe. The possibility of an increasing G has been suggested by several authors.

We would like to mention here that Beesham (1994), Lima and Carvalho (1994), Kallingas et al. (1995) and Lima (1996) have also derived the Bianchi type I cosmological models with variable G and Λ assuming a particular form for G. These models have some similarities with our model (23) in the cases (1) and (3) only. But our derived results differ from these models in the sense that the both of these are constrained by the equation of state whereas we have neither assumed equation of state nor particular form of G.

The behaviour of the universe in our models will be determined by the cosmological term Λ ; this term has the same effect as a uniform mass density $\rho_{eff} = -\Lambda/4\pi G$, which is constant in space and time. A positive value of Λ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of Λ, the expansion will tend to accelerate; whereas in the universe with negative value of Λ, the expansion will slow down, stop and reverse. Recent cosmological observations (Garnavich et al. 1998; Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 2004; Schmidt et al. 1998) suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, our models are consistent with the results of recent observations.

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